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1993 J. Phys. A: Math. Gen. 26 4081

(http://iopscience.iop.org/0305-4470/26/16/023)

J. Phys. A: Math. Gen. **26** (1993) 4081–4085. Printed in the UK

Electromagnetic fields produced by the spike pulse of hard radiation

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Received 13 August 1992, in final form 8 March 1993

Abstract. Electromagnetic fields produced by the spike pulse of hard radiation are discussed. We propose a model from which the electromagnetic field of a running pulse generated by a line current can be derived. The solution of the electrodynamic problem is found by the Smirnov method of incomplete separation of variables by means of the Riemann method.

While travelling through a medium, hard radiation produces charged particles. Atoms absorb quanta of radiation by the photoabsorption effect. The angular distribution of photoelectrons is proportionate to $1 + 4(v/c)\cos\vartheta$ where c is the velocity of light, v is the velocity of photoelectrons, ϑ is the angle, which is reckoned from the direction of quantum motion. Therefore, photoabsorption results in a partially regular motion of charged particles, and this current of photoelectrons produces an electromagnetic field.

By virtue of the fact that the source of the photoelectrons is the pulse of radiation, the front of the domain generating photoelectrons moves with the velocity of light. Thus the front of the resulting current pulse (but, of course, not the electrons themselves) propagates at light velocity. The track length of the radiation quanta is much larger than that of the photoelectrons, hence the current pulse is considered to be a line current.

The electromagnetic field produced by hard radiation has been considered earlier for cases when the approximation of the line current was not applicable [1, 2]. In turn, the line current pulse with the velocity of light has been taken as a source of the radiation of a line antenna [3]. The current in a particular point of the antenna is a function of time, reckoned from the moment when the front of the pulse arrives at this point. Here we are looking for the solutions where the current pulse is an arbitrary function of time and location.

The expression for the current density $\boldsymbol{j}$ in cylindrical coordinates ρ, φ, z is

$$\boldsymbol{j} = j_z \boldsymbol{e}_z$$

$$j_z = \frac{1}{2\pi} h(\tau - z) h(z) \frac{\delta(\rho)}{\rho} J(z, \tau) \qquad \tau > 0 \tag{1}$$

$$j_z \equiv 0 \qquad \tau < 0$$

where $J(z, \tau)$ is the total current, $\tau = ct$, $h(\tau - z)$ is the Heaviside function, and $\delta(\rho)$ is the Dirac function.

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0305-4470/93/164081+05$07.50

Components of the vector of electric field strength $\boldsymbol{E}$ and of magnetic induction $\boldsymbol{B}$ satisfy the equations

$$\begin{aligned} &\frac{\partial B_\varphi}{\partial \tau} = -\frac{\partial E_\rho}{\partial z} + \frac{\partial E_z}{\partial \rho} \qquad -\frac{\partial B_\varphi}{\partial z} = \frac{\partial E_\rho}{\partial \tau} \\ &\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho B_\varphi) = \frac{\partial E_z}{\partial \tau} + \frac{4\pi}{c} j_z. \end{aligned} \tag{2}$$

The initial conditions are

$$\boldsymbol{E} = \boldsymbol{B} \equiv 0 \qquad \tau < 0. \tag{3}$$

Let us try to find the solution of (2) in the form

$$E_\rho = \frac{\partial^2 u}{\partial \rho \partial z} \qquad E_z = -\frac{\partial^2 u}{\partial \tau^2} + \frac{\partial^2 u}{\partial z^2} \qquad B_\varphi = -\frac{\partial^2 u}{\partial \tau \partial \rho}.$$

The function u is the Bromwich–Borgins potential [4, 5] which is generally used for electromagnetic waves in free space. Remarkably, it also holds for some special types of current distribution [6]. Substituting E_ρ, E_z, and B_φ into (2) one can prove that the first two equations are satisfied, and the third one together with the initial conditions (3) yield

$$\begin{aligned} &\frac{\partial^2 v}{\partial \tau^2} - \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial v}{\partial \rho}\right) - \frac{\partial^2 v}{\partial z^2} = \frac{4\pi}{c} j_z \\ &v \equiv 0 \qquad \text{for } \tau < 0 \end{aligned} \tag{4}$$

where $v = \partial u / \partial \tau$.

The φ-component of magnetic induction is

$$B_\varphi = -\partial v / \partial \rho. \tag{5}$$

The solution of problem (4) can be found by the Smirnov method of incomplete separation of variables [7], and subsequent solution of the equation depending on the cordinate z and time τ with the help of the Riemann formula.

We use the Fourier–Bessel transform

$$f(s, z, \tau) = \int_0^\infty \rho \, \mathrm{d}\rho \, f(\rho, z, \tau) J_0(s\rho)$$

$$f(\rho, z, \tau) = \int_0^\infty s \, \mathrm{d}s \, f(s, z, \tau) J_0(s\rho)$$

where J_0 is the Bessel function of the first kind of order zero. From (1) one obtains

$$\begin{aligned} &\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial z^2} + s^2\right) v(s, z, \tau) = \frac{4\pi}{c} j_z(s, z, \tau) \\ &j_z(s, z, \tau) = \frac{1}{2\pi} h(\tau - z) h(z) \, J(z, \tau) \qquad \text{for } \tau > 0 \\ &j_z(s, z, \tau) \equiv 0 \qquad \text{for } \tau < 0 \end{aligned}$$

while the second part of (4) yields

$$v(s,z,\tau) \equiv 0 \qquad \text{for } \tau < 0.$$

The last result leads to the homogeneous initial conditions for the Cauchy problem

$$v = 0 \qquad \partial v/\partial\tau = 0 \qquad \text{for } \tau = 0 \qquad z \in (-\infty, +\infty).$$

We can obtain the unique solution of this problem using the Riemann method [8] (the Riemann function is $J_0(s[(\tau-\tau')^2 - (z-z')^2]^{1/2})$

$$v(s,z,\tau) = \frac{1}{c}\int_0^\tau \mathrm{d}\tau' \int_{\tau'+z-\tau}^{-\tau'+z+\tau} \mathrm{d}z'\, h(z')h(\tau'-z')J(z',\tau')J_0(s[(\tau-\tau')^2 - (z-z')^2]^{1/2}).$$

Then the solution of problem (4) can be represented by the integrals

$$v(\rho,z,\tau) = \frac{1}{c}\int_0^\tau \mathrm{d}\tau' \int_{\tau'+z-\tau}^{-\tau'+z+\tau} \mathrm{d}z'\, h(z')h(\tau'-z')J(z',\tau') \times \int_0^\infty s\,\mathrm{d}s\, J_0(s\rho)J_0(s[(\tau-\tau')^2 - (z-z')^2)]^{1/2}). \tag{6}$$

If the current pulse has finite duration T, we can denote it in (1) explicitly

$$j_z(\rho,z,\tau) = \frac{1}{2\pi}h(\tau - z)h(z)h(T-\tau+z)\frac{\delta(\rho)}{\rho}J(z,\tau) \qquad \tau > 0.$$

In this case integrand in (6) contains the additional factor $h(T-\tau'+z')$.

Let the pulse of current have a duration $T = \infty$. Substituting

$$\int_0^\infty s\,\mathrm{d}s\, J_0(s\rho)J_0(s[(\tau-\tau')^2-(z-z')^2]^{1/2}) = \frac{1}{\rho}\delta(\rho-[(\tau-\tau')^2-(z-z')^2]^{1/2})$$

into (6) we obtain the following

$$v = \frac{1}{c\rho}\int_0^\tau \mathrm{d}\tau' \int_{\tau'+z-\tau}^{-\tau'+z+\tau} \mathrm{d}z'\, h(z')h(\tau'-z')J(z',\tau')\delta(\rho - [(\tau-\tau')^2 - (z-z')^2]^{1/2}). \tag{7}$$

From relation (7) we have the solution of (4)

$$v = \frac{1}{c}\int_0^{\tau-(\rho^2+z^2)^{1/2}} \mathrm{d}\xi_1'\, \frac{1}{\tau - z - \xi_1'} J\left(\xi_1', \tau + z - \frac{\rho^2}{\tau - z - \xi_1'}\right) \tag{8}$$

where the total current is taken as a function of $\xi_{1,2}' = \tau' \mp z'$ and the last argument ξ_2' must be replaced by $\tau + z - \rho^2/(\tau - z - \xi_1')$. The solution is non-zero within the sphere of radius τ centred at the origin of the coordinates where the current pulse starts, and the wavefront equation is $\tau = (\rho^2 + z^2)^{1/2}$. One can obtain the latter result by simple geometric treatment using the causality principle.

The solution of (8) can be generalized in the case when the pulse of current has duration $T \neq \infty$. If the time of observation τ is less than $T + (\rho^2+z^2)^{1/2}$, the field is described by expressions (5), (8). If $\tau > T + (\rho^2+z^2)^{1/2}$, then the upper limit in the integral (8) is T.

Let the line of current have a finite length l and the duration of the pulse be infinite ($T \to \infty$). Again, one can denote it explicitly. In this case the integrand (6) contains the additional factor $h(l - z')$. For the time of observation $\tau < l + (\rho^2 + (z-l)^2)^{1/2}$ the solution (8) holds; for $\tau > l + (\rho^2 + (z-l)^2)^{1/2}$ we have

$$v = \frac{1}{c}\int_{\tau-l-(\rho^2+(z-l)^2)^{1/2}}^{\tau-(\rho^2+z^2)^{1/2}} \mathrm{d}\xi_1' \frac{1}{\tau - z - \xi_1'} J\left(\xi_1', \tau + z - \frac{\rho^2}{\tau - z - \xi_1'}\right). \tag{9}$$

The only difference between (8) and (9) is the lower boundary of the integral which corresponds to the wave which is emitted in the current track end ($\rho = 0$, $z = l$) at $\tau = l$.

Let the current pulse have finite duration T. For $\tau > l + (\rho^2 + (z-l)^2)^{1/2}$ and $T > \tau - l - (\rho^2 + (z-l)^2)^{1/2}$ the integral in (9) has the upper limit T; if $T < \tau - l - (\rho^2 + (z-l)^2)^{1/2}$ then $v = 0$.

Formulas (8), (9) are correct for a δ-pulse of current.

Now we consider the fields generated by a spike pulse of hard radiation in some simple cases. Let the hard radiation be absorbed by an inhomogeneous medium. If the total current can be approximated as

$$J(z, \tau) = \Phi(z)\mathrm{i}(\tau - z) \qquad z \in (0, \infty)$$

then relation (8) gives

$$v = \frac{1}{c}\int_0^{\tau-(\rho^2+z^2)^{1/2}} \mathrm{d}\xi_1' \frac{1}{\tau - z - \xi_1'} \mathrm{i}(\xi_1')\Phi\left(\frac{1}{2}(\tau + z - \frac{\rho^2}{\tau - z - \xi_1'} - \xi_1')\right). \tag{10}$$

If $\mathrm{i}(\xi_1') = \delta(\xi_1')$, we obtain, with the use of (5)

$$B_\varphi = \frac{1}{c}\frac{1}{\tau - z}\left(\frac{\rho}{(\rho^2 + z^2)^{1/2}}\Phi(0)\delta(\tau - (\rho^2 + z^2)^{1/2}) + h(\tau - (\rho^2 + z^2)^{1/2})\frac{\rho}{\tau - z}\frac{\partial}{\partial S}\Phi(S)\right)$$

$$S = \frac{\tau^2 - \rho^2 - z^2}{2(\tau - z)}.$$

The second term contains information about the absorption of radiation by the atoms of the medium and hence about the properties of the latter.

Let the hard radiation be absorbed by the atmosphere of the earth. The source of the radiation is at a height H, the frequency of the hard radiation is ν. The distribution of photoelectrons is proportionate to the spectral intensity of the radiation and if $J(z, \tau) = \Phi(z)\delta(\tau - z)$ then

$$J(z, \tau) = \Phi(0)\delta(\tau - z)\exp(\alpha z - (\varkappa_\nu(H)/\alpha)(\exp(\alpha z) - 1))$$

where α is the barometric constant, $\varkappa_\nu(H)$ is the coefficient of absorption at the height of the source, and $\Phi(0)$ is a constant. The radiation propagates to the earth surface. Substituting $J(z, \tau)$ into (10), we obtain

$$B_\varphi = \frac{1}{c}\Phi(0)\left\{\frac{\rho}{(\tau - z)(\rho^2 + z^2)^{1/2}}\delta(\tau - (\rho^2 + z^2)^{1/2}) - h(\tau - (\rho^2 + z^2)^{1/2})\frac{1}{\tau - z}\frac{\partial}{\partial\rho}\right.$$
$$\left.\times \exp\left[\alpha\frac{\tau^2 - \rho^2 - z^2}{2(\tau - z)} - \frac{\varkappa_\nu(H)}{\alpha}\left(\exp\left(\alpha\frac{\tau^2 - \rho^2 - z^2}{2(\tau - z)}\right) - 1\right)\right]\right\}.$$

Let the radiation be absorbed in a bounded region and the length of the current line be l. Let the total current of the photoelectrons be $J(z,\tau) = J(\tau - z)$. In particular, this corresponds to the model of full ionization of a homogeneous medium. With the help of (5), (8), (9) we can obtain

$$B_\varphi = \frac{\rho}{c}\Phi(0)\left\{h(\tau-(\rho^2+z^2)^{1/2})\frac{1}{(\rho^2+z^2)^{1/2}((\rho^2+z^2)^{1/2}-z)}J(\tau-(\rho^2+z^2)^{1/2})\right.$$
$$-h(\tau-l-(\rho^2+(z-l)^2)^{1/2})\frac{1}{((\rho^2+(z-l)^2)^{1/2})((\rho^2+(z-l)^2)^{1/2}+l-z)}$$
$$\left.\times J(\tau-l-(\rho^2+(z-l)^2)^{1/2})\right\}.$$

The relation obtained is in agreement with the results of [3].

If the absorbed energy density of the hard radiation can be described as a continuous function, $z \in (0,l)$, and the total current of photoelectrons can be approximated as $J(z,\tau) = \Phi(z)\delta(\tau - z)$, $\Phi(0) = \Phi(l) = 0$, we obtain from (5), (8), (9) the following expression

$$B_\varphi = \frac{\rho}{c}\frac{\rho}{(\tau-z)^2}\frac{\partial\Phi(S)}{\partial S}[h(\tau-[\rho^2+z^2]^{1/2})-h(\tau-l-[\rho^2+(z-l)^2]^{1/2})]$$
$$S \in (0,l) \qquad \tau \neq z.$$

The factor $\partial\Phi/\partial S$ is proportional to the gradient of the absorption energy density.

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